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LETTER TO THE EDITOR

Test of universality for percolative diffusion

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Abstract. We study the influence of several modifications on models of diffusion on a randomly disordered lattice. We find that magnetic field, inertia and myopic behaviour do not change the universality behaviour whereas electrical field and interaction between the walker and next neighbouring lattice sites appear to produce at least one new universality class.

The problem of classical diffusion on random percolation clusters has been studied for several years (de Gennes 1976, Straley 1980, Gefen *et al* 1983, Mitescu and Roussenq 1983, Pandey *et al* 1984). In this paper we investigate the effects of nearest-neighbour interaction with the lattice and the influence of an external electrical and magnetic field using Monte Carlo methods. We observed the behaviour of the exponent k in the predicted relation $R \propto t^k$ (Mitescu and Roussenq 1983) and also of the exponent m in the relation $R \propto (p_c - p)^{-m}$. The investigations of Pandey *et al* (1984) will be continued for the myopic and for the looking ant.

The influence of an electrical field has already been studied theoretically by Barma and Dhar (1983) and also by Böttger and Bryskin (1982). For a strong influence of the field they made different predictions. Barma and Dhar expected that for a certain bias the drift velocity would vanish, whereas Böttger and Bryskin predicted that even for a high bias the drift will be reached over long times. In this question we will extend the Monte Carlo simulations carried out by Pandey (1984).

In all cases the diffusing particle may only stay on an occupied site of a lattice, where each site is occupied randomly with a probability p.

The program we used for our investigations was developed by Pandey *et al* (1984), containing a realisation of the well known blind ant model (Mitescu and Roussenq 1983) and was modified by us (Suessenbach 1984, Seifert 1984). For various times t we measure the euclidian displacement $R(t)$ from the local origin. To get reliable values for $R(t)$ we average over a large number of ants and over a sufficiently large number of lattices. The exponent k is evaluated with a least square fit of the $\log(R(t))$–$\log(t)$ plot or in the case of the electrical and magnetic field just by evaluating the gradient of the straight line between the two points $\log(t_i)$ and $\log(t_{i+1})$, where $t_i = 2^{i-1}$ and $i > 0$.

To study different time scaling and nearest neighbour interactions Suessenbach (1984) examined two other types of ants.

(1) *Myopic ant.* It chooses with equal probability one of the directions leading to an accessible site. We realise this by choosing a new target site with the help of the random generator if the respective target is non-occupied without increment of the

time counter. In this way the myopic ant can be interpreted as a blind ant with different time scaling.

(2) *Looking ant.* It chooses the direction to jump according to the following criterion: (a) the site should be accessible (b) the transition probability is proportional to Q^N, where N is the number of accessible neighbouring sites of the site to which the jump attempt is made and Q is a given parameter simulating the interaction strength.

These types of ants can have the additional property 'inertia', where the ant chooses with probability $\frac{1}{2}$ the same direction as in the previous step and with probability $\frac{1}{10}$ one of the other five directions.

Seifert (1984) studied the other influences of an electrical and a magnetic field applied to the blind ant model. 'Electrical field' means that the ant chooses one direction with a higher probability than the others. This higher probability is determined by the factor $0 < B < 1$, so that the direction to jump is chosen with the probability $B + (1 - B) \times \frac{1}{6}$ along the field and each of the other five directions with $(1 - B) \times \frac{1}{6}$.

The other property 'magnetic field' means, that the ant chooses with a certain probability, determined by the factor $0 < F < 1$, the direction clockwise perpendicular to the previous step, if the previous step led to an occupied site and was made perpendicular to the field. Otherwise all directions are chosen with the same probability.

Most of the simulations were done on the Cyber 205 Vector Computer in Bochum, which is up to 13 times faster than the scalar Cyber 76 Computer. We let 512 ants diffuse simultaneously on simple cubic lattices up to a size 176^3 or on triangular lattices up to a size 2560^2 with helical boundary conditions, and with time steps up to 10^6 and 10^7. Typically, we averaged over 7–10 lattices.

The results were as follows.

Myopic ant

First we found that the mean visitation probability of every occupied site on every cluster at $p = p_c$ is proportional to the number of occupied neighbours of that site in the myopic ant case, while the blind ant model shows equal visitation probability for every site of a given cluster. That means, that apart from the different time scaling the myopic ant also shows a different microscopic diffusion behaviour. On the other hand we could show that there are no significant differences in the exponent k between the blind and myopic ant model.

The asymptotic exponent k was extrapolated for $1/R \to 0$ from figure 1, which is similar to the method applied by Pandey *et al* (1983). We found

$$k = 0.195\frac{+0.01}{55-0.02}.$$

In contradiction to the Boston results (Havlin *et al* 1984) we could not find any significant differences in the time the blind ant respectively the myopic ant used to reach their asymptotic value of k.

Looking ant

The looking ant algorithm required a processor time about ten times higher than the other ant types. Therefore we produced Monte-Carlo data for this model only up to 10^5 steps. Thus we could only determine an effective exponent k_{eff}. It was determined by a linear fit of 91 values of $R(t)$ with $t = j \times 1000$ and $j = 10, 11, 12, \ldots, 100$ for

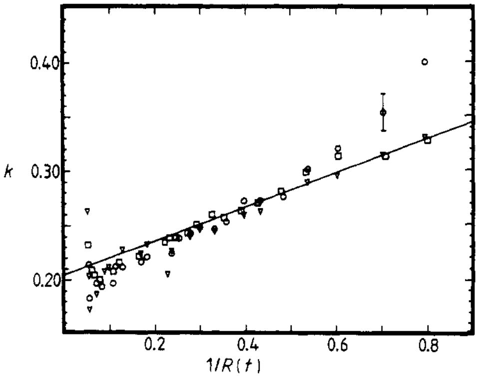

Figure 1. Exponent k as a function of the reciprocal RMS radius $R(t)$. Squares, myopic ant; circles, inertial myopic ant; triangles, inertial blind ant data. The straight line shows the extrapolated trend of the data of the blind ant model, published by Pandey *et al* (1984). All data produced at $p = p_c$ with lattice size 160^3.

different interaction energies $\log(Q)$ and $p = p_c$. The results of this procedure can be seen in figure 2.

Using an extreme interaction parameter ($Q = 0.001$) we could see two oscillating phenomena of $R(t)$: a short-time oscillation with period $t = 1$ and a nearly constant amplitude of 0.08 and a long-time oscillation with exponentially increasing period (the first of length about $t = 10^4$) and an amplitude near 0.1. The short-time oscillation points to uniform behaviour of the majority of the ants on a large number of different lattices at $p = p_c$. They seem to be driven into local 'traps' by the extreme interaction, getting out at the next step due to their myopic property, driven back again at the next step and so on. The long-time oscillations appear to be the same as those we found in the biased ant case at extreme bias values. They may be related to the so-called trapping effects caused by the fractal character of the Cluster shape at $p = p_c$ (Pandey 1983, Barma and Dhar 1983 and Dhar 1984).

Analysing the present data, it is not yet clear whether the looking ants present one new universality class independent of Q, i.e. $k_{\text{eff}} \to 0$ for $t \to \infty$, or whether we found a

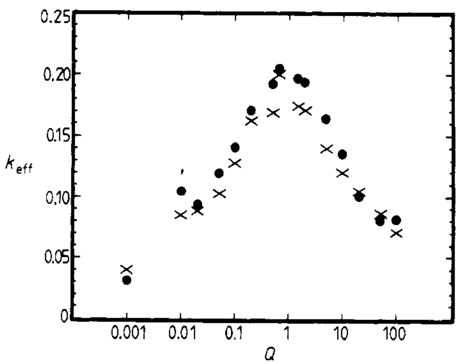

Figure 2. Effective exponent k_{eff} as a function of the interaction strength Q at $p = p_c$ and lattice size 160^3. Circles, looking ant; crosses, inertial looking ant model.

continuum of different universality classes (each one fixed by a special value of the interaction strength Q).

Inertial ants

Within our error bars we could show that the inertial ant types have the same asymptotical behaviour as the non-inertial models. We found:

$$\text{Inertial blind ant: } k = 0.19\begin{matrix}+0.01\\-0.02\end{matrix}$$

$$\text{Inertial myopic ant: } k = 0.19\begin{matrix}+0.01\\-0.02.\end{matrix}$$

The effective exponents of the inertial looking ants can be seen from figure 2. They show the same behaviour for different values of Q as the exponents of the non-inertial looking ants. All these reults lead to the assumption that inertia does not change the universality behaviour of the observed diffusion models.

The exponent m

The blind ant model, which requires the least processor time (218 ns/step), was used to produce data for the saturation value R^2_∞ for $p < p_c$. Near to p_c R^2_∞ diverges with

$$R^2_\infty \propto (p_c - p)^{-m}$$

where m is predicted to be $m = 2\nu - \beta = 1.34 \pm 0.02$ for three-dimensional simple cubic lattices. Mitescu and Roussenq (1983) found $m = 1.65 \pm 0.05$ in agreement with earlier studies, which would lead to a contradiction with scaling assumptions. Pandey *et al* (1984) found $m = 1.2$. We produced, using nearly 50 000 seconds of CYBER 205 processor time, 24 values of R^2_∞ for different $(p_c - p)$ from $p_c - p = 0.05$ up to $p_c - p = 0.16$ with lattice size 160^3 and tried to fit the power law

$$R^2_\infty = a(p_c - p)^{-m}(1 + b(p_c - p)^\Delta).$$

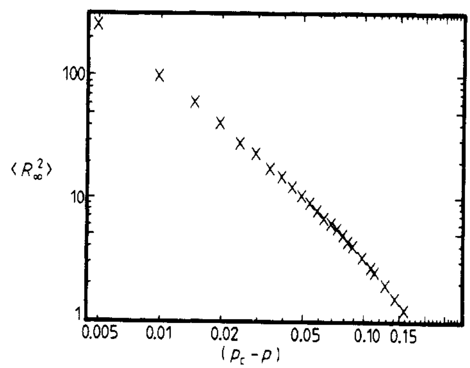

Figure 3. Saturation value of R^2_∞ as a function of $(p_c - p)$. This data was produced with lattice size 160^3.

We found

$$m = 1.32 \pm 0.13$$

$$\Delta \sim 1.0, \qquad a \sim 0.25, \qquad b \sim 1.0$$

which seems to confirm the basic scaling assumptions and which also seems to be an improvement over the less accurate results of Mitescu and Roussenq and of Pandey *et al* respectively.

Electrical field on a simple cubic lattice

We studied the behaviour of the exponent k and also the exponent $k_\|$ and $k_\perp$ for the distances parallel and perpendicular to the electrical field.

These simulations were done at $p > p_c$, where the required processor time for a single (350 ns/step) is about 60% higher than in the case of the unbiased blind ant. We found for a small bias a faster growth of the distance R dependent on time t and for a higher bias a slower growth (even slower than in the case of no field) which is due to the trapping effect (Barma and Dhar 1983, Pandey 1983, Seifert 1984). For a small bias we also observed that the exponent k approaches the value 1 asymptotically. We got the same result for the exponent $k_\|$, whereas the exponent $k_\perp$ approaches 0.5.

In view of the contracting predictions of Barma and Dhar on one hand and Böttger and Bryskin on the other hand, we investigated biased diffusion for a high bias factor B and observed an oscillating character of the exponent. These oscillations were independent of the set of random numbers used and the lattice size. For times up to 10^6 the exponent did not appear to approach $k = 1$, but for times up to 10^7 it did, as one can see at least in the case $B = 0.99$, $p = 0.725$ (figure 4). These results seem to be in accordance with Böttger and Bryskin, although we are unable to explain the systematic oscillations, which are not statistical fluctuations or finite size effects. We also do not yet know if there is a connection to the oscillations that are theoretically predicted by White and Barma (1984). For further investigations it would be interesting to find out if the exponent after reaching the value 1 decreases again or if it stays at the value 1. Instead of using a field in direction of one coordinate we also used a field in the (111)-direction (Pandey 1984). In contrast to the case of bias in direction of one coordinate we observed a maximum of k at $t \sim 10$ instead of a minimum and a minimum at $t \sim 100$. After that, so for $t > 100$, we found almost identical behaviour of the exponent in both cases.

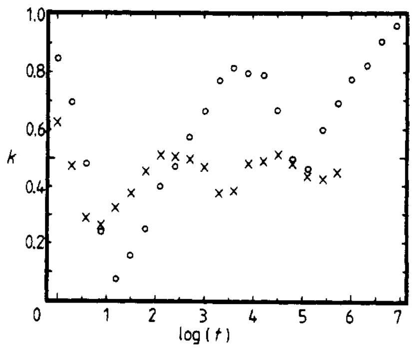

Figure 4. Exponent k as a function of $\log_{10}(t)$ for $p = 0.725$, $B = 0.99$ (circles) and $p = 0.5$, $b = 0.9$ (crosses) on a simple cubic lattice sized 176^3.

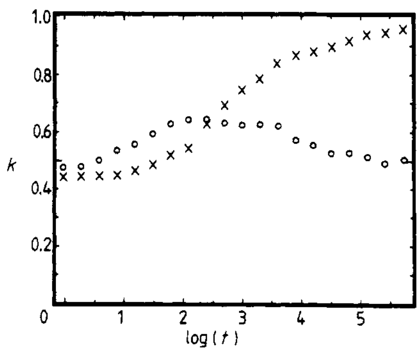

Figure 5. Exponent k as a function of $\log_{10}(t)$ at $p = 0.6$ on a triangular lattice sized 2560^2 with $B = 0.1$ (crosses) and $B = 0.3$ (circles).

Electrical field on a triangular lattice

These investigations were done at $p > p_c$. For small bias we got the same results for the behaviour of the exponents as in the case of the simple cubic lattice; that k and $k_\parallel$ approach 1, whereas $k_\perp$ approaches 0.5. For a higher biasfactor ($B = 0.3$ at $p = 0.6$) we observed that the exponent increases its value, but only reaches 0.64 and then decreases again (figure 5). It could be that this is only the beginning of the oscillating character that the exponent showed on a simple cubic lattice at $p = 0.725$. But it could also mean that Barma and Dhar are right, that the drift velocity vanishes for a certain bias, even though we could not find their predicted behaviour of the drift velocity (Seifert 1984).

Influence of a magnetic field on a simple cubic lattice

These simulations were carried out on 70 lattices containing 176^3 sites at $p = p_c$, where the time for a single step of the ant is 430 ns. We found that for a small magnetic field (small F) the growth of R is faster than in the case of no field. A simple explanation for this behaviour was given by Gefen at the International Topical Conference on Kinetics of Aggregation and Gelation in Athens USA in April 1984 (Seifert 1984). The behaviour of the exponent k under influence of a magnetic field seems to be unchanged, so that the influence of a magnetic field does not change the universality class of the random walk.

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